Limitations on Mass Separation by the Weakly Ionized Plasma Centrifuge

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In weakly ionized argon and xenon rotating plasmas the rotational velocity and the temperature and pressure distribution have been measured.

The stationary discharge is generated by two opposed cathode-anode configurations. The arc current of 100 A is drawn across an axial magnetic field up to 0.26 T. The filling pressure is varied between 1 and 10 torr.

The rotational velocity is found to be proportional to the discharge current and the magnetic field and inversely proportional to the viscosity of the neutral gas. The rotational kinetic energies of the particles in the argon and xenon discharge are about equal. Because the temperature of the argon discharge is lower than that of the xenon discharge, the pressure rise in radial direction due to centrifugal forces is steeper for the former.

A theoretical analysis taking into account viscous dissipation as the only heating mechanism yields a heavy particle temperature T which imposes an upper limit to the ratio $X = \frac{1}{2} m v_\theta^2/kT$ of order unity. The maximum attainable separation factor α is therefore limited in these types of centrifuges. Experimentally, in the parameter region studied, X is found not to exceed a value 0.4 in argon discharges and 0.2 in xenon discharges. A rough estimate shows that besides viscous dissipation other heating mechanisms are also important. Ohmic heating, for instance, is at least a factor 6 larger than the viscous dissipation.

1. Introduction

Since about 1965 plasma centrifuges have been investigated as a possible means to separate isotopes (Bonnevier [1], James and Simpson [2], Boeschoten [3], Nathrath [4], Kaneko [5]).

In this type of experiments the rotation of the particles is caused by the Lorentz force which is present when an electric current flows across a magnetic field (Lehnert [6]).

In our case we deal with a stationary weakly ionized plasma inside a cylindrical vessel (Wijnakker et al. [7]). Due to the applied axial magnetic field and the radial component of the electrical current the ionized particles are driven in azimuthal direction. Via collisions with the ions, the neutrals are set in rotation in the same direction. The equilibrium separation factor with respect to the composition at the axis of a homogeneously rotating medium is $\alpha = \exp(\Delta m\, v_\theta^2/2\,k\,T)$, where Δm is the mass difference of the particles, v_θ the azimuthal velocity, k the Boltzmann constant and T the absolute temperature (Bonnevier [8]).

Our experiment differs from other weakly ionized plasma centrifuges in that a double cathode-anode configuration is used which provides a large central region in the device where axial gradients of the rotational velocity and plasma potential are small. In an earlier paper (Wijnakker et al. [7], hereafter called WGK) measurements of the rotational velocity of ions and neutrals, plasma potential and pressure rise were given for a stationary rotating argon plasma at a filling pressure p_F of 1 torr (p_F is measured at the wall).

In this paper measurements of the rotational velocity of the neutral particles, the heavy particle temperature and the radial pressure enhancement are presented for argon and xenon discharges as a function of filling pressure and magnetic field strength.

The results are compared with the one-fluid M.H.D. theory also applied in WGK [7]. Attention is also paid to the energy equation, which gives information about the heavy particle temperature in the system. This is an important parameter because it influences the pressure rise in the system and the separation power of the centrifuge. In Sect. 2 the experimental set up is described. The results of the measurements are given in Section 3. The theoretical explanation of the results is presented in Sect. 4 together with some implications of the solution of a part of the energy equation.

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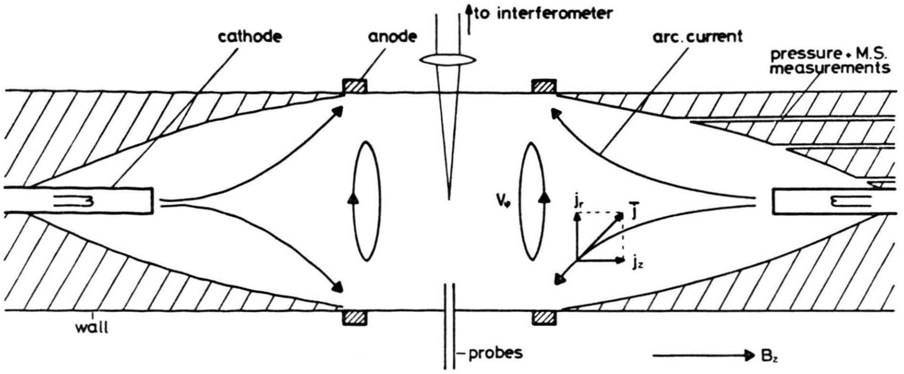

Fig. 1. Schematic view of experiment. The cylindrical vessel contains two cathodes and two ringshaped anodes. The radial part of the electric current density j_r and the applied axial magnetic field B_z are responsible for the azimuthal velocity v_θ of the particles.

2. Description of the Experiment

2.1. Apparatus

A schematic view of the experiment is given in Figure 1. The cylindrical vessel consists of 24 electrically insulated stainless steel rings, two of which serve as anodes: The length between the two cathodes is 48 cm, the diameter in the mid plane is 12 cm. The distance between the anodes is 14 cm. For a more extensive description of a similar device see WGK [7]. The tips of the two tungsten cathodes are heated partly by plasma ion bombardment and partly by (indirect) heating from the inside by electron bombardment. The diameter of the cathodes is 1.5 cm. Maximum currents of 100 A are drawn from the cathode to the ring-shaped anode in the wall. A homogeneous axial magnetic field with a strength up to 0.26 tesla is applied externally.

All measurements are performed in the symmetry plane between the two anodes. The velocity and temperature of the plasma are determined from the Doppler shift and broadening respectively of spectral lines of the plasma light; in both cases a pressure-scanned Fabry-Perot interferometer is used.

The pressure rise in radial direction is determined by means of tantalum pressure probes which are designed such that only the static pressure is measured (WGK [7]).

2.2. The Discharge Mode

The plasma discharge is operated in the arc regime. Different modes of the discharge are possible (Love and Park [9]). In our case there is one mode in which the discharge is more or less uniformly spread out over the whole interior of the vessel. In this so-called diffuse mode an appreciable rotational velocity and radial pressure enhancement are found to be present between the two anodes. Therefore this mode seems to be best suited for the investigation of separation effects.

The degree of ionization in this mode in the mid plane between the anodes is estimated to be lower than 10% in all cases. The motion of the electrons can be considered to be collisionless, $\beta_e \gg 1$, and that of the ions collisional, $\beta_i \leqq 1$. β_e and β_i represent the electron and ion Hall parameter, respectively (WGK [7]). In this regime the Hall effect is found to be small.

This diffuse mode can only exist for specific combinations of the arc current, the axial magnetic field and the pressure in the vessel. Figure 2 gives the parameter region in which this mode is present in 100 ampere argon and xenon discharges. At low values of the pressure the plasma ion bombardment of the cathodes is not sufficient to sustain the discharge (lowest horizontal line). The maximum magnetic field strength applied is 0.26 tesla (the dashed vertical line). At the hypotenuse of the triangle the discharge mode suddenly changes character. The diffuse mode passes into another one in which secondary flow effects become more important (Van den Berg [10]). The discharge seems to break into two separate ones; the intensity of the spectral lines under study decreases by more than one order of magnitude. Occasionally this second mode is also found to exist in the parameter region of the diffuse mode. In this mode the velocity of the neutral particles in the mid plane of the system is low and there is little pressure enhancement in radial direction (Figure 6). For this reason this second mode is considered to be of less importance as far as the mass separation is concerned.

3. Results

In this chapter measurements of the rotational velocity, temperature and pressure are given in

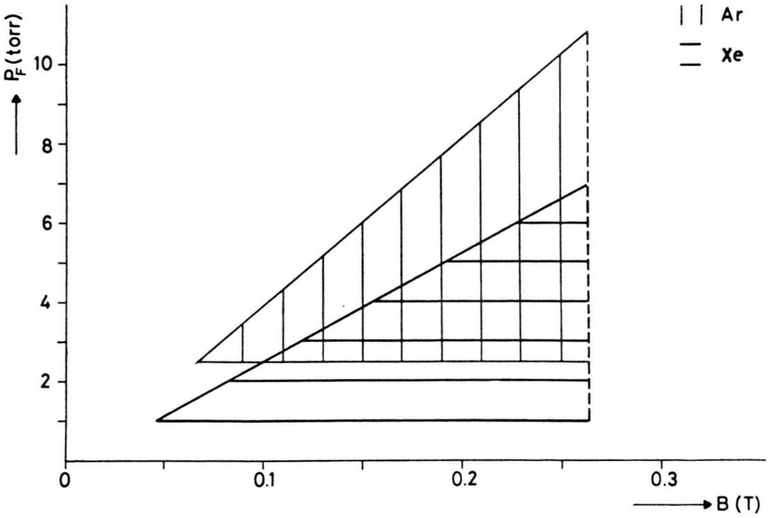

Fig. 2. Parameter region in which the argon and xenon discharges can be maintained in the diffuse mode. In this mode all measurements have been done. At high pressure and low magnetic fields another discharge mode is present in which much lower rotational velocities are measured in the mid plane of the experiment.

argon and xenon discharges. The measurements have been done in the diffuse mode at different values of the filling pressure and magnetic field strength. The electric current is 100 A in all cases. For the optical measurements mainly spectral lines of the neutral particles are used. Owing to the low degree of ionization, the fluid velocity and temperature are predominantly determined by the neutral particles.

The laterally observed spectral line profiles have not been subjected to an Abel transformation. Because in this diffuse mode the intensity of the measured spectral lines is nearly constant over the radius (WGK [7]), the not-corrected velocities for both the argon and xenon discharges are too low $(30-40\%)$. Without this transformation the temperature can be determined accurately only in the centre of the mid plane where the rotational velocity is zero. Outside the centre the combination of Doppler broadening and Doppler shift leads to errors which can exceed a factor of two (Kress [11]). Due to the intergration along the line of sight the measured values of the temperature in the centre of the column are estimated to be about 30% lower than the actual ones. Because the theoretical analysis presented in Sect. 4 requires only relative rotational velocities and temperatures, we present the measured values without the above mentioned corrections. Additional broadening due to Zeeman splitting is eliminated by inserting a polarisation filter in the light path. In the argon discharge the 4702 Å neutral line and the 4807 Å ion line are

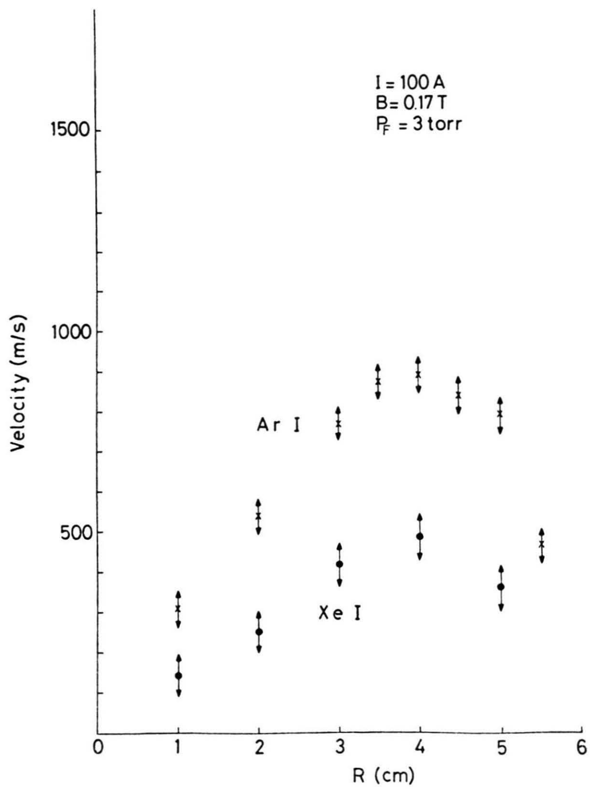

Fig. 3. A typical example of the measured azimuthal velocity of argon neutrals in an argon discharge and xenon neutrals in a xenon discharge at different radial positions. The electric current is 100 A, the magnetic field strength 0.17 T and the filling pressure is 3 torr. The argon neutrals rotate about a factor 1.7 faster than the xenon neutrals. The maximum velocity in both discharges is reached about 4 cm from the axis.

used; in the xenon discharge the 4806 Å neutral line.

3.1. Rotational Velocity

In Fig. 3 a typical example is shown of a velocity profile measured at different radial positions for one particular combination of pressure, magnetic field strength and discharge current.

The maximum velocity of neutrals both in an Ar and in a Xe discharge is reached at about 4 cm from the centre of the cylinder. Over the range of $R = 0-3$ cm the plasma rotates like a rigid body (constant angular velocity). The maximum velocity of the Ar neutrals is about 1.7 times higher than that of the Xe neutrals.

In Fig. 3 and all following figures the relative accuracy for one measuring series is given by the error bars. The reproducibility of the results is of the order of 15%. Within the accuracy of the measurements the shape of the velocity profile is found to be independent of the discharge parameters; therefore the rest of the paper deals with the maximum values.

Figure 4 a gives the maximum velocity of the neutrals in an argon and a xenon discharge as measured for different values of the magnetic field strength at a filling pressure of 3 torr and an electrical current of 100 A. The velocity increases approximately linearly with the magnetic field strength. The highest measured velocity of the argon neutral particles is 1350 m/s; for the xenon particles it is 825 m/s. At all magnetic field strengths the ratio of the velocities of Ar en Xe neutrals is equal to 1.7 ± 0.2. Because Ar is 3.3 heavier than Xe, the rotational kinetic energies ($\frac{1}{2}\, m\, v_\theta^2$) of Ar en Xe are equal within the accuracy of the measurements.

Figure 4 b shows the maximum velocity as measured for different filling pressures. The mag-

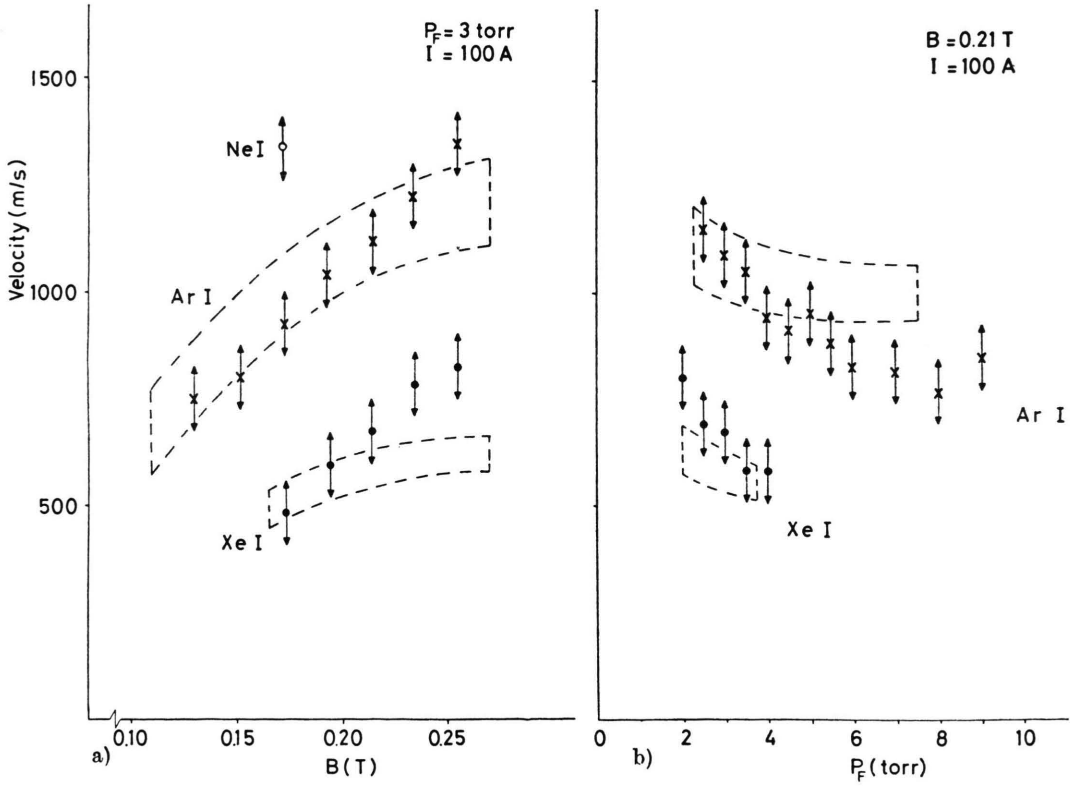

Fig. 4. The measured maximum azimuthal velocity of the argon and xenon neutrals as function of the magnetic field strength B (4a). The current I is 100 A and the filling pressure 3 torr. For neon the discharge is difficult to maintain in the diffuse mode. Therefore only one point is given in the figure. For the three discharges $\frac{1}{2} m\, v_\theta{}^2$ is about equal for the same value of B.
The dependence of the velocity on the filling pressure is given in 4b. $I = 100$ A and $B = 0.21$ T. For each discharge a dashed area is given in which the values for the velocity calculated according to $v_\theta \sim IB/\mu$ should be found; μ is the coefficient of viscosity (for details see text).

netic field strength is kept constant at 0.21 T and the electric current at 100 A. The velocity of the neutrals decreases with increasing filling pressure. In the argon discharge a pressure increase from 3 to 9 torr leads to a velocity decrease of a factor 1.5.

3.2. *Heavy Particle Temperature*

In Fig. 5 a the temperature in the centre of argon and xenon discharge is given as a function of the magnetic field strength at a filling pressure of 3 torr and a current of 100 A. The temperature increases slightly at higher values of the magnetic field strength in both discharges.

Ions and neutrals have about the same temperature. The heavy particle temperature in the Xe discharge is a factor 2.1 ± 0.1 higher than that of the Ar discharge. Maximum temperatures measured in the Ar and Xe discharges are 12 800 and 25 800 K, respectively.

The temperature in the discharge is practically independent of the filling pressure (Figure 5 b). In the case of Xe the temperature seems to increase somewhat at higher pressures.

If we determine the ratio of the rotational kinetic energy $\frac{1}{2} m v_\theta^2$ at the velocity maximum and the thermal energy $k T$ at the centre for various combinations of I, B and p_F we find values for xenon and argon which do not exceed 0.2 and 0.4 respectively. These ratios probably will be slightly larger if also $k T$ is taken at the position of the velocity maximum. Note that in argon gas as well as xenon gas the sound velocities are reached when $\frac{1}{2} m v_\theta^2 / k T = 0.83$.

3.3. *Pressure Distribution*

In Fig. 6 an example is given of the radial pressure distribution in some typical argon and xenon discharges. In the diffuse mode the pressure probe

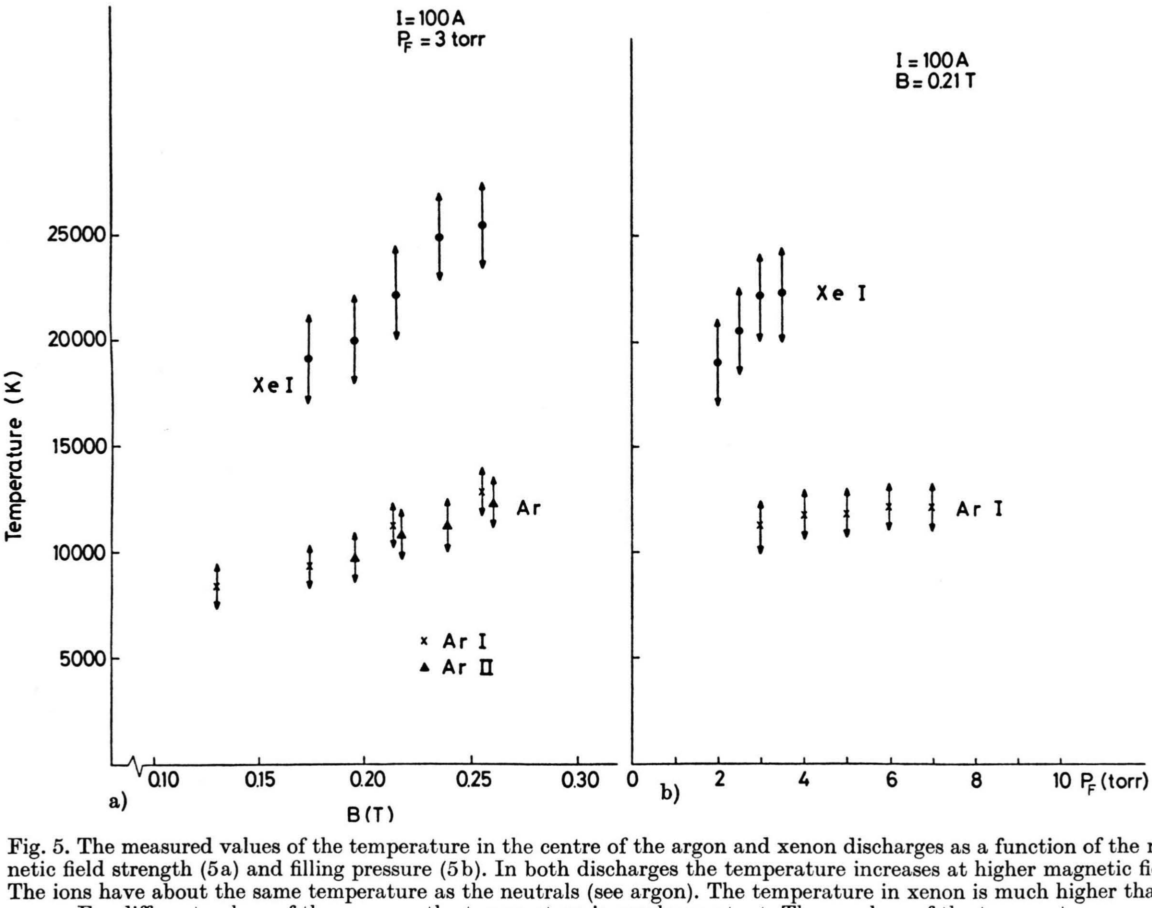

Fig. 5. The measured values of the temperature in the centre of the argon and xenon discharges as a function of the magnetic field strength (5a) and filling pressure (5b). In both discharges the temperature increases at higher magnetic fields. The ions have about the same temperature as the neutrals (see argon). The temperature in xenon is much higher than in argon. For different values of the pressure the temperature is nearly constant. These values of the temperatures are used for the calculation of the viscosity μ (see Fig. 4a and 4b).

can only be operated over a region of 0.7 cm inwards from the wall because of the risk of melting the probe.

In both discharges there is a pressure enhancement near the wall. The pressure rise appears to be higher in the argon discharge than in the xenon discharge. In the second mode there is hardly any pressure rise; also the temperature is so low that the probe can be shifted inwards up to about 4 cm from the centre.

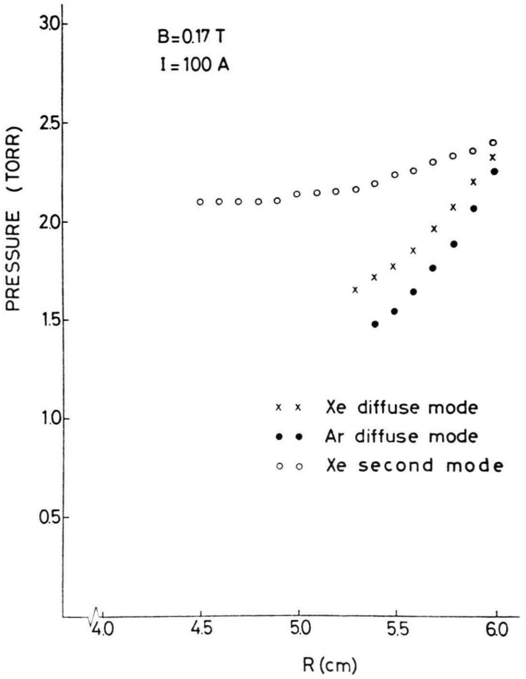

Fig. 6. The measured pressure as a function of the radius in an argon and xenon discharge. $B = 0.17$ T and $I = 100$ A. In the diffuse mode only from the wall up to 0.7 cm inwards can be measured due to the high temperatures. The pressure rise is steeper in argon than in xenon. In the second mode (see text) the temperatures are lower and practically no pressure rise is present.

4. Theoretical Analysis

4.1. *Velocity and Viscosity*

The one-fluid M.H.D. theory developed by Klüber [12] and Wilhelm and Hong [13] appears to describe the plasma behaviour in crossed electric and magnetic fields reasonably well; the agreement between the measured velocities and plasma potentials and the calculated values is found to be good (WGK [7]).

In this theory the azimuthal component of the Navier-Stokes equation gives the relation between the rotational velocity v_θ, the coefficient of viscosity μ, the radial electric current density j_r and the axial magnetic field strength B_z:

$$\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_\theta)\right)+\frac{\partial^2 v_\theta}{\partial z^2}\right]=j_r B_z\,. \quad (1)$$

In the present experiment where σ, the electrical conductivity parallel to the magnetic field, is much larger than $\sigma_\perp$, the conductivity perpendicular to the magnetic field, the rotational velocity is almost independent of the axial (z) coordinate, especially in the vicinity of the mid plane ($z=0$). Consequently the term $\partial^2 v_\theta/\partial z^2$ is expected to be much smaller than the other terms in Equation (1).

Under these circumstances the current distribution depends weakly on z and therefore for a given radial position (r) we can write $j_r \sim I$ in which I is the total arc current. This finally leads to the conclusion that the maximum velocity will be proportional to the quantity $I B/\mu$. This is confirmed by a numerical analysis of the system which shows that this is true for an effective Hall parameter $\beta_{\text{eff}} \geqq 8$ (see Fig. 5 of WGK [7], where β_{eff} is to be defined according to $\sigma_\perp = \sigma/(1+\beta_{\text{eff}}^2)$; it contains contributions from electron as well as ion conduction (Mitchner and Kruger [14]).

The coefficient of viscosity μ in principle represents the total fluid viscosity and therefore contains contributions from electrons as well as ions and neutral particles. However, the contribution from the electrons is negligible (Tannenbaum [15]). The contribution from the other plasma particles to the total fluid viscosity is roughly inversely proportional to their respective collision cross sections (Smirnov [16]). This together with the fact that the plasma is weakly ionized means that the viscosity of the system is mainly determined by neutral-neutral collisions. Therefore the coefficient of viscosity of the plasma can be taken equal to that of the neutral gas. At equal temperatures the viscosity of an argon gas is nearly equal to that of a xenon gas. At a temperature of 7000 K for instance, the viscosity of argon is about 10% lower than that of xenon (Dymond [17]). For both gases the coefficient of viscosity increases roughly with temperature T according to $\mu \sim T^{3/4}$. From the viscosity tables of Dymond and the Figs. 5 a and 5 b the coefficients of viscosity can be determined as a function of the magnetic field strength B and the filling pressure p.

Consequently the product $I\,B/\mu$ can be calculated as a function of B and p for argon as well as for xenon. Due to the relative inaccuracy of the temperature measurements the value for $I\,B/\mu$ lies between two extreme values. In Figs. 4 a and 4 b, the dashed areas represent the calculated dependence of $I\,B/\mu$ on B and p. In both figures all calculated values for $I\,B/\mu$ are normalized on the argon rotational velocity at $B = 0.21$ T and $p = 3$ torr. It should be mentioned that no extra normalization on the measured Xe velocities is applied; in principle relation (1) is mass independent!

From the relative behaviour of the measured velocities and the "$I\,B/\mu$ areas" it can be concluded that the dependence of the measured velocity on B and p is slightly stronger than can be accounted for by the proportionality with $I\,B/\mu$. However, the general behaviour seems to be described correctly, especially as far as the difference in velocity between argon and xenon is concerned.

The radial pressure enhancement is given by the radial component of the Navier-Stokes equation (Wilhem and Hong [13], WGK [7]):

$$\frac{\partial p}{\partial r} = \frac{m\,v_\theta^2}{k\,T} \cdot \frac{p}{r} \,. \tag{2}$$

Because $m\,v_\theta^2$ is approximately equal for Ar and Xe while the temperature of the Ar discharge is lower than that of the Xe discharge, relation (2) predicts the lower radial pressure rise for the heavier of the two noble gases. This somewhat surprising result is confirmed by the experimental data shown in Figure 6.

For similar reasons as given for the coefficient of viscosity also the coefficient of thermal conductivity λ is mainly determined by that of the neutral particles. The thermal conductivity of Xe gas is appreciably lower than that of Ar. According to Dymond [17] at for instance 7000 K the thermal conductivity of Ar gas is by a factor of 2.8 higher than that of Xe. The much higher heavy particle temperature of the Xe discharge could be related to this fact.

4.2. *Viscous Heating*

High rotational velocities and low heavy particle temperatures are favourable for a good separation result. However the velocity of the particles is apparently coupled with the heavy particle temperature (see Figures 4 a and 5a). To get some insight in the possible relation between velocity and temperature it is necessary to solve the energy equation which describes the thermal balance of the system. In the stationary case the energy equation of the heavy particles (i. e. ions and neutrals) is given by

$$\frac{3}{2}\,\boldsymbol{v}\,\nabla p + \frac{5}{2}\,p\,\nabla\,\boldsymbol{v} + \boldsymbol{\pi} : \nabla\boldsymbol{v} + \nabla\boldsymbol{q} = Q_{\mathrm{ei}} + Q_{\mathrm{en}} + Q_{\mathrm{R}} \,, \tag{3}$$

where $\boldsymbol{\pi}$ is the viscosity tensor and q the heat flow density vector. The first two terms on the l.h.s. represent the convective heat flow, the third one the viscous heating, the fourth one the heat flow by thermal conduction. On the r.h.s. the term Q_{R} gives the Ohmic heating due to the part of the current carried by the ions. Equation (3), being a heavy particle energy equation, does not contain the Ohmic heating of the electrons nor radiation losses of the medium; the influence of the electrons on the heavy particle temperature is taken into account via the terms Q_{ei} and Q_{en} which represent the energy transfer by collisions from electrons to ions and neutrals, respectively. For Q_{ei} and Q_{en} we can write (Mitchner and Kruger [14]):

$$Q_{\mathrm{ei}} = 3\,n_{\mathrm{e}}\,\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}\,v_{\mathrm{ei}}\,k\,(T_{\mathrm{e}} - T_{\mathrm{i}}) \,, \tag{4}$$

$$Q_{\mathrm{en}} = 3\,n_{\mathrm{e}}\,\frac{m_{\mathrm{e}}}{m_{\mathrm{n}}}\,v_{\mathrm{en}}\,\mathrm{k}\,(T_{\mathrm{e}} - T_{\mathrm{n}}) \,. \tag{5}$$

In relations (4) and (5) n_{e} is the electron density; the quantities m_{e}, m_{i} and m_{n} the electron, ion and neutral mass; ν_{ei} and ν_{en} the collision frequencies between electrons and ions and electrons and neutral particles, T_{e} the electron temperature, T_{i} and T_{n} the ion and neutral particle temperature. Due to the experimental fact that the ions have the same temperature as the neutral particles (Fig. 5 a), it suffices to consider only one heavy particle temperature.

We will solve (3) for the mid plane of our experiment ($z = 0$), where all first derivation with respect to the axial coordinate z are zero. Besides that, as was mentioned before (WGK [7], these derivatives are small everywhere else between the two anodes. Therefore in first order all second derivatives to z are assumed to be zero. Besides that, as was done in previous calculations, we suppose secondary flows to be absent i. e. radial and axial components of the velocity are zero. The convective terms now disappear and (3) becomes for q equal to $-\lambda \nabla T$:

$$\boldsymbol{\pi} : \nabla \boldsymbol{v} + \nabla \boldsymbol{q} = -\mu \left(r \frac{\partial}{\partial r} \frac{v_\theta}{r} \right)^2 - \nabla (\lambda \nabla T) = Q_{\mathrm{ei}} + Q_{\mathrm{en}} + Q_{\mathrm{R}} \,. \qquad (6)$$

In good approximation we can take $\lambda \sim T^{3/4}$ (Dymond [17]). Equation (6) now becomes

$$\frac{-\partial^2 T}{\partial r^2} + \frac{-1}{r} \left(\frac{\partial T}{\partial r} \right) + \frac{-3}{4} \frac{1}{T} \left(\frac{\partial T}{\partial r} \right)^2 = \frac{\mu}{\lambda} \left\{ r \frac{\partial}{\partial r} \frac{v_\theta}{r} \right\}^2 + \frac{Q_{\mathrm{ei}} + Q_{\mathrm{en}}}{\lambda} + \frac{Q_{\mathrm{R}}}{\lambda} \,. \qquad (7)$$

In (7) the quantity μ and λ are both temperature dependent, however the ratio μ/λ is independent of the temperature (Hirschfelder [18]); for neutral mono-atomic gases it is given by

$$\frac{\mu}{\lambda} = \frac{4}{15} \frac{m}{k} \,. \qquad (8)$$

Due to the fact that viscous heating is directly linked to the velocity profile, it is inherently present in this kind of plasma centrifuges. For that reason we will concentrate on the influence of viscous heating on the temperature distribution, and in first instance not take into account both collision terms and the Ohmic heating term. In that idealized case the solution of (7) gives an absolute lower limit of the calculated temperature. Inclusion of the terms Q_{ei}, Q_{en} and Q_{R} will always enhance this temperature. This enhancement depends strongly on the experimental conditions.

The temperature profile can be calculated if an azimuthal velocity profile is known (Equation (7)). If we normalize the velocity profile to a maximum velocity $v_{\theta \mathrm{m}}$, then v_θ can be written as $v_\theta(r) = v_{\theta \mathrm{m}} V(r)$ in which $0 \leqq V(r) \leqq 1$.

If Q_{ei}, Q_{en} and Q_{R} are taken zero, Eq. (7) becomes with (8)

$$\frac{-\partial^2 T}{\partial r^2} + \frac{-1}{r} \left(\frac{\partial T}{\partial r} \right) + \frac{-3}{4} \frac{1}{T} \left(\frac{\partial T}{\partial r} \right)^2 = \frac{4}{15} \frac{m v_{\theta \mathrm{m}}^2}{k} \left\{ r \frac{\partial}{\partial r} \frac{V(r)}{r} \right\}^2 . \qquad (9)$$

Of course this equation only holds for a weakly ionized plasma in which the viscosity and the heat conduction are predominantly determined by the neutral gas.

For the solution of (9) we will take two velocity profiles. The first one $v_\theta \sim |r \ln r|$ and the second $v_\theta \sim J_1(k_1 r)$, see Fig. 7a $(0 \leqq r \leqq 1)$. The first profile is a theoretical one which is obtained when in an infinitely long cylinder a radial current is drawn across an axial magnetic field from a vanishingly thin plasma column at the axis to the cylinder wall. The second profile follows from earlier results (WGK [7]) in which the velocity profile in the symmetry plane between the anodes can be described by a summation of an infinite series of first order Bessel functions. In the mid plane the first term of this series provides the major contribution to the total sum. Near the centre $(0 \leqq r \lesssim 0.2)$ this profile is equivalent to that of a rigid rotator (angular velocity v_θ/r is constant) which means that the viscous heat production is zero there (Figure 7b). The first velocity profile $v_\theta/r \sim \ln r$ gives rise to a viscous dissipation (divided by λ) (see (9)) which is constant over the radius. If we choose $\frac{1}{2} m v_{\theta \mathrm{m}}^2 = 3.32\, 10^{-20}$ J for both profiles, which for argon is equivalent to a maximum velocity of 1000 m/s, a numerical solution of (9) yields temperature profiles shown in Fig. 8 (Bakker [19]). In both cases the temperature at the wall is assumed to be 400 K. As can be expected the temperature is strongly dependent on the velocity profile. Because for the separation of species of different mass the relation between the kinetic energy $(\frac{1}{2} m v_\theta^2)$ and the thermal energy $(k T)$ is important also the quantity $\frac{1}{2} m v_\theta^2/k$ is plotted in Figure 8. For both velocity profiles the calculated values of $\frac{1}{2} m v_\theta^2/k$ are for a given radial position (r) at most of the order of the temperature; i.e. $\frac{1}{2} m v_\theta^2/k T \lesssim 1$. For a given velocity profile the quantity $\frac{1}{2} m v_{\theta \mathrm{m}}^2/k T$ is found to be constant and therefore independent of $v_{\theta \mathrm{m}}$. Note that in a mechanical ultra centrifuge in which v_θ/r is constant there is no viscous heating.

Due to the fact that in these weakly ionized rotating plasmas the rotational kinetic energy is already of the same order of magnitude as the thermal energy even in the case that only viscous heating is taken into account, the pressure rise and therefore the separation factor α will be limited. In a Xe discharge, for instance, in which the isotopes have a highest mass number difference of 12, for $\frac{1}{2} m v_{2 \mathrm{m}}^2/k T = 1$, the separation factor is at most 1.09. For uranium a maximum separation factor of 1.013 is found.

It should again be mentioned that this result is only valid for the idealized case in which the axial gradients are small and no secondary flows are present. In case secondary flows are as-

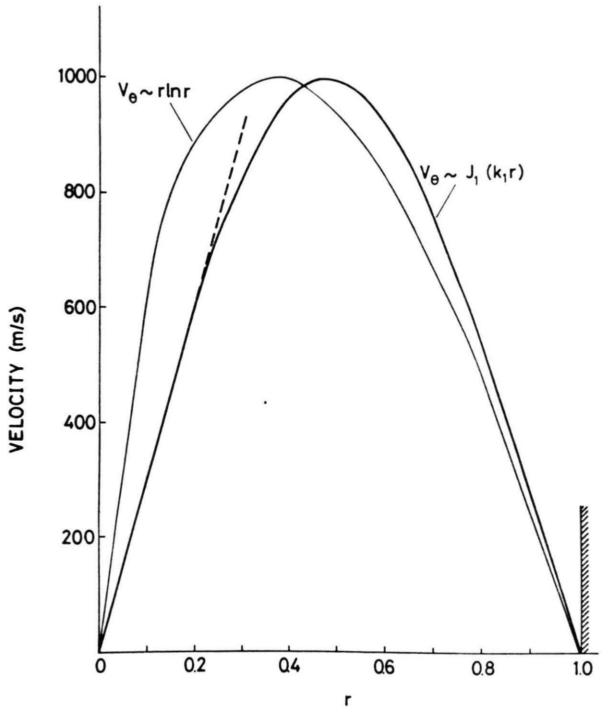

Fig. 7. a) The two velocity profiles chosen for the calculation of the temperatures in the rotating plasma system. $v_\theta \sim r \ln r$ and $v_\theta \sim J_1(k_1 r)$. Near the centre the second profile is similar to that of a rigid body rotator (v_θ / r = constant, see dashed line). Both profiles are normalized to a maximum velocity of 1000 m/s.

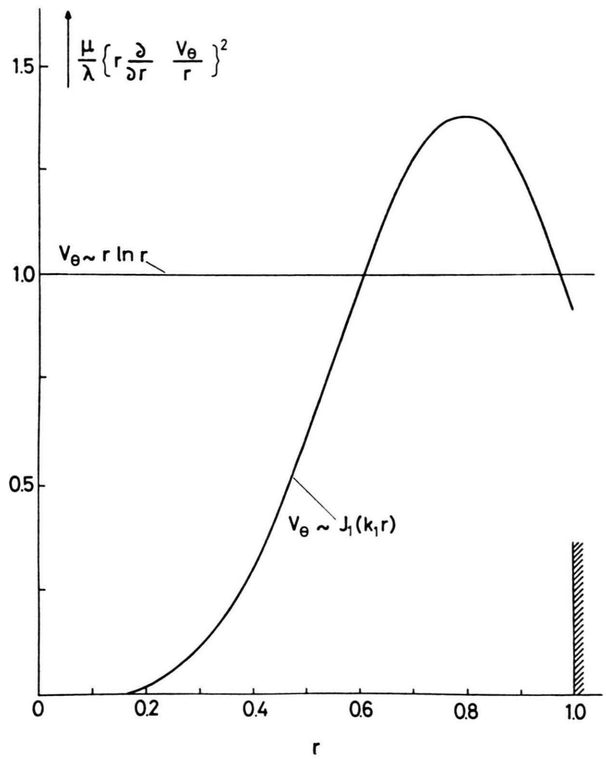

Fig. 7.b) The viscous heating term

$$\mu\left(r\frac{\partial}{\partial r}\frac{v_\theta}{r}\right)^2 = \frac{4}{15}\,\frac{m\,v_{\theta\mathrm{m}}^2}{k}\left(r\frac{\partial}{\partial r}\frac{V}{r}\right)^2$$

as it appears in eqs. (7) and (9). The horizontal profile, indicated by $v_\theta \sim r \ln r$, is normalized to one. Both profiles are calculated for the same maximum velocity and are given on the same scale that a velocity profile $v_\theta \sim J_1(k_1 r)$ leads to a viscous heating which is zero close to the centre.

sumed to play a role, convective cooling takes place; gas from the hot core then flows along the water cooled walls. However these secondary flows may disturb the separation. This means that there will probably be an optimum situation in which the ratio $\frac{1}{2}m v_\theta^2/kT$ and consequently the separation factor α might be (slightly) larger than the values given above (v. d. Berg [10]).

As was pointed out before, viscous dissipation is not the only source of heat in this type of systems, it gives only a lower limit of the temperature. Heating due to collisions with the electrons, Q_{ei} and Q_{en}, and Ohmic heating Q_{R} can also play an important role. This is clear from the measured temperatures (Figs. 5 a and 5 b) which are much higher than the temperatures found theoretically when only viscous heating is taken into account. Moreover, for equal values of $\frac{1}{2}m v_{\theta\mathrm{m}}^2$, as found experimentally for Ar and Xe, a calculation based on viscous heating alone leads to equal temperatures for both gases. This is certainly not the case; the xenon discharge is much hotter than the argon one.

A rough estimate of the magnitudes of viscous heating, Ohmic dissipation and of both collision terms Q_{ei} and Q_{en} can be obtained as follows. In an argon discharge in which the particles rotate according to $v_\theta \sim |r \ln r|$ and $v_{\theta\mathrm{m}} = 1000$ m/s, the viscous heating is equal to 1.6 kW/m^3 (for μ we take the value of $2.2 \cdot 10^{-4}$ kg m^{-1} sec^{-1} at $T = 7000$ K (0.6 eV); see Dymond [17]). Note that the viscous dissipation is proportional to v_θ^2 and $\mu(\sim T^{3/4})$. If we assume that the radial current is fully carried by the ions (WGK [7]) the Ohmic heating term $Q_{\mathrm{R}} = j_\perp^2/\sigma_\perp$, in which $j_\perp$ is the radial current density. If we choose two electron temperatures $T_{\mathrm{e}} = 1$ and 2.5 eV, which in this case corresponds to $\beta_{\mathrm{eff}} = 3$ and 9 and $\sigma_\perp = 121$ and 50 Ω^{-1} m^{-1}; see WGK [7] and Section 4.1., Q_{R} is equal to about 9 and 50 kW/m^3, respectively. Note that the total input

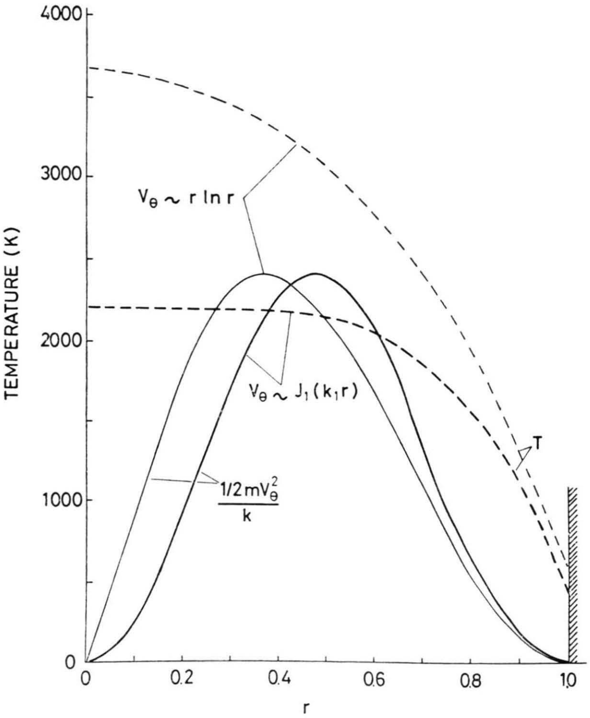

Fig. 8. The temperature profiles calculated from the energy equation in case only viscous heating is taken into account. The velocity profiles given in Fig. 7a are used for the calculation. The wall temperature is kept constant at 400 K. A maximum rotational kinetic energy $\frac{1}{2}m\ v_{\theta m}^2 = 3.32 \times 10^{-20}\ J$ is chosen for both profiles (for argon this means $v_\theta = 1000$ m/s).
Because the relation between the rotational kinetic energy ($\frac{1}{2} m\ v_\theta^2$) and the thermal energy ($k\,T$) is important for the mass separation, also the quantity $\frac{1}{2} m\ v_\theta^2/k$ is given for both velocity profiles. Note that in both cases $\frac{1}{2} m\ v_\theta^2/k\,T \lesssim 1$.

power for a discharge current of 100 A is on average 120 kW/m³ so heating powers exceeding this value are impossible.

Both the collisional heating terms Q_{ei} and Q_{en} are zero in case the electron temperature T_e is equal to the heavy particle temperature T. The heating due to Q_{ei} and Q_{en} very rapidly becomes large for T_e larger than T. For instance in case of a filling pressure of 1 torr, a degree of ionization of 5%, a heavy particle of 10 000 K (0.91 eV), electron temperatures equal to $T_e = 1.0$ and 2.5 eV lead to $(Q_{ei} + Q_{en}) = 50$ and 150 kW/m³, respectively. It is clear that under these conditions it is unlikely that in the experiment T_e exceeds T by a large amount. It is also clear that viscous heating probably plays a minor role. However, although difficult to achieve, the other heating mechanism can possibly be reduced by choosing proper experimental conditions whereas viscous heating is inherently present.

5. Conclusions

In our weakly ionized plasma centrifuge there is one discharge mode, called the diffuse one, which is well suited to study the possibilities for separation of species of different mass. This mode can be maintained within certain values of the parameters p, B and I.

The dependence of the measured rotational velocity of the neutrals on current, magnetic field strength, temperature and mass can be described by relation $v \sim I\,B/\mu$ in which μ is the coefficient of viscosity of the neutral particles. Under equal experimental conditions the neutrals of argon in an argon discharge rotate faster than those of xenon in a xenon discharge. This is due to higher temperatures in xenon and therefore higher values for the coefficient of viscosity. The temperature difference is probably due to a difference in the coefficient of thermal conductivity which is mass dependent.

For the same values of the electric current, magnetic field and filling pressure, the rotational kinetic energies $\frac{1}{2}\ m\ v_\theta^2$ of the Xe and Ar particles are about equal. Due to the fact that the temperatures are different in both discharges, this should give rise to a higher radial pressure rise in argon which indeed is confirmed experimentally.

A solution of the energy equation in which only the viscous dissipation as a heat source is taken into account and in which secondary flows are neglected, indicates that the temperature in the discharge is coupled with the rotational velocity. Because of viscous dissipation higher velocities give rise to higher temperatures such that $\frac{1}{2}\ m\ v_\theta^2/k\,T \lesssim 1$. Unless cooling mechanisms can be found which do not affect the rotational velocity this fact alone limits the maximum attainable value for the separation factor α in this kind of plasma centrifuges. Calculations show that for instance Ohmic heating is probably even more important than viscous heating, in agreement with the measurements which give much higher temperatures than predicted if only viscous heating is taken into account. In the parameter region studied the experimentally determined ratio $\frac{1}{2}\ m\ v\ /k\,T$ is found to be $\lesssim 0.2$ for xenon and $\lesssim$ 0.4 for argon.

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